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Assembling efficient organizations?

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Abstract

This essay is a variation on a theme[1] by Arrow (1974), that the limits of organization are set by the need to economize on information flows, thought, committee meetings, and infighting. It is scored for the instruments I can play: duality, profit functions, and, above all, aggregates.

Introduction

Why *aggregates*? Macroeconomics was invented by Keynes for the management of natural economies. For that, most economists agree, one would like to deal in terms of them. Is that a reasonable possibility? Aggregation theorists say no! The condition for them to work perfectly is that everyone should behave similarly at the margin: were there to be a single aggregate for equipment, for instance, a steel firm would change its production in the same way when given a new blast furnace as a typewriting agency given enough word processors.[2] That seems altogether too unrealistic for them to work reasonably well in practice.[3]

The general ideas developed here were discussed in a number of seminars, workshops, and conferences in the last decade, including Kenneth Arrow's on organization theory. I am grateful to him, Angus Deaton, James Mirrlees, and John Muellbauer for their comments.

[1] Of course, there are other themes. The difficulty of getting people to agree on a common aim is also important and can also be regarded as a problem in aggregation. The profit centres that emerge from the analysis may be interpreted in this light.

[2] Enough to increase its profits, at the given prices, by the same amount as the blast furnace would the steel firm's. Remember, both are gifts.

[3] That is, assuming all the firms do not march step in step, investing and disinvesting together, for instance. Tom Stoker (1982, 1984), in particular, explored cases in which this is not so, as had Henri Theil (1954). One does not need many

God did not make the world, then, for the benefit of macroeconomists.

It is not God but businessmen who make our large firms, commonly by taking others over, breaking them up, and selling off the pieces they do not want. Presumably they do so for their own benefit and, in particular, with an eye to ease of control. If countries would be easier to manage through broad aggregates, were this not misleading, would not large firms be, too, and would not businessmen bear this in mind in deciding what to take over and what to sell off?

It would only be one goal, of course, to be traded off against others, and only important if realistic. Hence the '?' in the title.

Are appropriate components likely to exist?

The condition that all the firms in a country should behave similarly at the margin requires them to be too alike. Its implications for individual firms are more acceptable. They run in terms of *overheads, variable costs* that are constant per unit over a range of output, *activities* running at levels determined by their relative profitabilities and the equipment available, and, given appropriate convexity, *profit centres.* These, moreover, are consequences of the theory, not assumptions, and the technologies in question are the only ones for which they work perfectly.[4] They are the terms that accountants, statisticians, industrial consultants, and businessmen use, presumably because they fit their experience. The chances are, then, that individual plants commonly are more or less of the appropriate type; the predatory businessman's role is to search out those that are sufficiently similar to each other or to those he already has.[5]

In what sense should the plants be 'similar'?

The activities correspond to the aggregates. The implications of similarity is that all firms use the same activities.[6] In the case of capital, the scale at which the corresponding activity runs depends only on the capital equipment it has[7]—similarly for land. In the case of consumer durables, for instance, its *only* outputs are consumer durables, although actual activities use labour and other current inputs—that

degrees of freedom to get the results I have quoted, and it is rather easy to see what sort of changes lead to the use of aggregates being badly misleading.

[4] Remember, however, that equipment and the like affect only the capacities at which the individual activities are run, not how they are.

[5] As it turns out, by using the same activities.

[6] At given prices, each activity produces current goods, net, in the same proportions wherever it is used—in the same amounts, indeed, per unit of capacity.

[7] Indeed, it is the most convenient measure of that firm f's *capital* or of its excess over that corresponding to a base endowment $\overline{u}_f$.

is, as negative outputs. The latter, at least, is unrealistic. However, industrialists need not think in this way. If they have assembled a number of factories, each using the same activities, into a large firm, it will be quite natural for them to think in terms of the levels at which those activities run, as this theory will predict. In that case, as we will see, each activity may produce or use all the current goods and services, whereas the scale at which it operates depends on its relative profitability and the fixed inputs available. The different plants may vary markedly in their overheads and in the efficiency with which they use their machines to produce 'capacity', but given the capacity available to an activity, it operates in the same way in different plants.

Two peculiarities remain.

First, there is the asymmetry between fixed and current inputs. The former affect only the scale at which the various activities operate, not the manner. This reflects in part the history of aggregation theory, initially focused on the differences between capital and other factors. However, the arguments use fixed inputs as shift parameters, to isolate particular firms, and to compare their behaviour at the margin. This can be avoided,[8] but at the cost of returning to aggregates specific to particular classes of goods. Its disadvantage here is that the managers of individual factories presumably know the state of their equipment better than those at the centre; its advantage is that the latter should presumably decide where investment should go.

Second, economists commonly believe in decentralization via prices, not target quantities. In fact, this is what happens here. The important macro variables turn out to be the profitabilities of the different activities, price aggregates dual to the scale variables whose value central management has been assumed to set. In fact, it turns out to be more sensible for them to estimate these profitabilities, pass them on to the factory managers, and leave it to each of them to choose the levels at which the various activities should operate in his own factory.

That cancels out the first peculiarity. Note that this, too, is a conclusion, not an assumption.

Two final admissions need to be made. First, ease of control is only one goal, to be traded off against others. This really is a case for bounded rationality, not optimization. Nevertheless, conditions for optimization are what economists know about and often give useful clues for the wider problem. Here, too, one of the goals is monopoly. Now, efficient monopolies presumably do better than inefficient ones and are more likely to last. For efficiency one needs common shadow

[8] See Gorman (1982) and the last paragraph of §1.

prices.[9] These replace the perfectly competitive prices of classical aggregation theory, arguably with more justification.[10]

Second, my modeling is distinctly lax. I fix the number of control variables and ask what the operating units should be like for these controls *not to reduce* their efficiency, that is, *not to cause costs to be set against the gains in the collection, dissemination, and processing of information assumed inherent in hierarchical control*, not to speak of possibly monopoly profits. I do not model the central office at all, although the analysis would hold were its costs to depend only on the values of the control variables.[11]

In particular, therefore, I seek perfect aggregates, or control variables, although I am really interested in those that perform reasonably well. The hope, as usual, is that the condition for this is that those for perfect aggregates hold approximately. Since my ultimate aim is approximate, however, I will be content with local results[12] and will feel free to use calculus and rely on the implicit function theorem. (I have not used it since my college days, so that the argument is distinctly clumsy.)

1. Heuristics

Aggregation theorists have traditionally dealt with the short run,[13] in which the *endowments*,

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[9] Of course, the goods have to be carefully defined: if the market for some goods is divided and some factories are only allowed to sell in one part, some in another, we would have to distinguish between those goods according to their market.

[10] However, monopolies are pretty robust even when inefficient and may even be defended against takeover bids by monopoly commissions and the like. One should remember Sir John Hick's dictum that the prime gain from monopoly is an easy life for the managers—perhaps not the sort of easy life I have been sketching.

[11] Of the form $\psi(p, v)$, where p is the price vector for current goods, and that of the target set by the centre. If so, the net demand for i by the central office would be $\psi_i(p, v) = \partial\psi/\partial p_i$ under weak conditions, given v, and this can be subtracted from the right-hand side of equation (1.9).

[12] Not approximate. Aggregates that work perfectly in a neighbourhood of the point in question. The neighbourhood may cover the whole space considered.

[13] Charles Blackorby, in particular, has recently published interesting papers on aggregation in intertemporal models (e.g. 1982), and both he (1984) and Frank Fisher (1982) on long-run aggregation. I refer to some unpublished work of my own on the latter below; however, this work does not really affect the point at issue.

$$u = (u_f)_{f \in F}\,, \tag{1.1}$$

of fixed inputs are taken as given, where u_f is the *fixed input*, or *endowment, vector* for the firm f; the *net production vector*,

$$x = \sum_{f \in F} x_f\,, \tag{1.2}$$

of current goods is chosen from the *short-run production possibility set*, or *technology*,

$$R(u) = \sum_{f \in F} R^f(u_f)\,, \tag{1.3}$$

whose simple additive structure, reflecting the absence of external economies and diseconomies, is the source of all the results to date and, in particular, explains their close family resemblance. This additive structure is preserved in the *gross profit function*

$$g(p,u) = \sup\{p \cdot x | x \in R(u)\} = \sum_f g^f(p, u_f)\,, \tag{1.4}$$

in an obvious notation, where p is a vector of *efficiency prices*, and in the *production plan*

$$x = g'(p,u) = \sum_f g'^f(p, u_f)\,, \tag{1.5}$$

where the prime denotes the price gradient[14] of the convex function in question. Since we are dealing with the short run, it is reasonable to assume that the technologies are closed strictly convex bodies, which implies that the gradients in question exist.

Traditional aggregates, whether for capital, land, semi-skilled labour, or consumer durables and whether scalars or vectors, can all be presented as quantities of intermediate goods; that is,

$$u \to v \to x\,, \tag{1.6}$$

so that v looks both ways, if you like, and particular structural assumptions are needed to identify particular components with one side rather than the other, assumptions that reflect the ideas of macroeconomists and are not appropriate here.

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[14] That is, the vector of price derivatives $g' := (g_i) = \partial g / \partial p_i$. Under the convexity condition, they exist throughout the interior of the function's domain, which is enough for our purposes.

Suppose now that $h(p,v)$ is the gross profit function of the *downstream technology* $v \to x$. Then

$$h(p,v) = g(p,u) = \sum_f g^f(p,u_f), \tag{1.7}$$

and hence

$$h'(p,v) = g'(p,u) = \sum_f g'^f(p,u_f), \tag{1.8}$$

either of which may be used to discuss particular problems in aggregation. Equation (1.8) was used to discuss the general problem in the version of this paper read at Kenneth Arrow's workshop in 1980. However, there is no reason to require that control variables be interpreted as intermediate goods. I will therefore revert to an earlier treatment (Gorman 1978) and require only that, given the prices p, they determine the production plan x; that is,

$$x = \theta(p,v) = g'(p,u) = \sum_f g'^f(p,u_f), \tag{1.9}$$

where $\theta(p,v)$ is not necessarily the gradient vector of a potential function $h(p,v)$,[15] much less of an admissible profit function.

In general,

$$v = \phi(p,u), \tag{1.10}$$

so that the central management might have to know the detailed efficiency prices p before making its decisions. One hopes that it will only need to know a few price indices, as indeed it will. In fact, it is these price indices, marginal profitabilities associated with the individual control variables, that turn out to be the valuable controls, allowing the central management to decentralize even broad planning to the individual factories in the short run and confine itself to gathering and processing information and to long-run planning. In fact, (1.10) becomes

$$v = \sum_f v_f = \sum_f \phi^f(a(p), u_f), \text{ say,} \tag{1.11}$$

at least locally, given reasonable smoothness, where

$$a(p) = (a^m(p))_{m \in M}. \tag{1.12}$$

[15] By a *potential function* $h(\cdot)$ I merely mean one for which $x = h'$, with no restriction as to shape.

Here M is a minimal set of controls, none of which is specific (see §2) to any one factory f, and $a^m(p)$ is the marginal profitability associated with the control m in a convenient normalization. Indeed, we can go further, for there exists a potential function[16]

$$k^f(a(p), u_f), \quad \text{for each } f \in F, \tag{1.13}$$

such that

$$v_f = k'^f(a(p), u_f), \tag{1.14}$$

where the prime represents the gradient with respect to the profitabilities a.

All these functions are conical in the prices or profitabilities, as one would like. When they are convex,[17] too, they correspond to perhaps fictitious technologies. If, for instance, $k^f(\cdot, u_f)$ is convex as well as conical, it corresponds to an *upstream technology* $S^f(u_f)$ for factory f, from which the factory's general manager chooses v_f to maximize profits $a \cdot v_f$ at the shadow prices a set by the central office. In other words, it is run as a profit centre. In general, however, $k^f(\cdot, u_f)$ is a potential function; the duty of the general manager is to know it and the state u_f of the factory's equipment and to calculate its gradient $k'^f(a, u_f)$ once he has been told the values of the a's.

That yields v_f. What next?

Here is another wonder. It turns out that

$$g^f(p, u_f) = k^f(a(p), u_f) - b^f(p), \quad \text{for each } f \in F, \tag{1.15}$$

so that

$$\begin{aligned} x_f = g'^f(p, u_f) &= \sum_m k_m^f a'^m(p) - b'^f(p) \\ &= \sum_m v_{fm} a'^m(p) - b'^f(p), \end{aligned} \tag{1.16}$$

in an obvious notation, by (1.14), so that

$$g^f(p, u_f) = p \cdot x_f = \sum_m v_{fm} a^m(p) - b^f(p), \tag{1.17}$$

and

$$g(p, u) = \sum_m v_m a^m(p) - b(p), \quad \text{say,} \quad v_m = \sum_f v_{fm}. \tag{1.18}$$

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[16] Strictly $k^f(\cdot, u_f)$, each $u_f \in U_f$, $f \in F$.

[17] Profit functions are closed convex conical and to each such function there corresponds a convex technology for which it is indeed the profit function.

See why I called the $a^m(p)$ marginal profitabilities? Note, too, that they yield net production patterns $a'^m(p)$ once the prices p are known. These net production patterns are called *activities*. They operate under constant returns, where v_{fm} is the level at which activity m operates in factory f. As we have seen, this level is decided by the general management of the factory in light of the profitabilities a passed on to them by central management. The job of the specialist manager of activity m in factory f is to know $a^m(\cdot)$ and the relevant prices p and then to calculate $a'^m(p)$ and produce $v_{fm}a'^m(p)$. If $a^m(\cdot)$ is convex as well as conical, and if we have normalized so that $v_{fm} \geq 0$, then there will be a technology A_m associated with $a^m(\cdot)$; the job of the specialist manager will be to choose x_{fm} from $v_{fm}A_m$ so as to maximize the profits $p \cdot x_{fm}$. Hence the specialist manager also would operate a profit centre.

Finally, the maintenance managers cover the overheads[18] as cheaply as possible.

The costs of central managers have not been explicitly modelled, so this is metatheory. At that level, their function is first to estimate the macro prices $a(p)$ for the coming period and then to pass their values to factory management. In the longer run, their function is to direct investment, research, and development and decide what to take over and what to sell off. The latter has already been discussed. To direct the flow of investment, estimates of future profitabilities $a(p)$ and of the general technological structure of their component factories *at the macrolevel*, as defined by $k^f(\cdot)$, are needed. The forms $a^m(\cdot)$ and $k^f(\cdot)$ also give a natural breakdown of research and development into *process research* (to make the individual activities more profitable) and that intended to squeeze more capacity out of existing equipment (to help different factories learn from each other by comparing their performance in that regard and to develop machines directed toward the more profitable activities).[19] Against this, the division between fixed inputs, determining, with $a(p)$, the scale at which activities operate, and the current production plans *within* these activities independent of the equipment available, seems unnatural and yet goes to the heart of the matter.

As mentioned in the Introduction, very similar results were derived by Gorman (1982) in the absence of fixed inputs, using appropriate

[18] $b^f(p) = -g^f(p, \overline{u}_f)$ in a convenient normalization, where $\overline{u}_f$ is a base endowment vector, and, therefore, closed concave conical and hence a true cost function.

[19] Metaphoric only. Of course, one can double the profits per unit in an activity by rescaling it.

prices rather than the endowments as the driving parameters, but only so far as the aggregates correspond to distinct classes of goods,[20] which I have already rejected as inappropriate in the present context.

2. The main result

The interest here is in local results and so calculus methods and, in particular, the implicit function theorem will be used. That turns on the rank,

$$\rho = \#M, \tag{2.1}$$

of control variables $m \in M$ needed, of the Jacobian matrix,

$$G(p,u) = [g_{is}(p,u)]\,, \tag{2.2}$$

involved, where

$$\begin{aligned} g_{is}(p,u) &= \partial^2 g/\partial p_i \partial u_s\,, \\ &= g^f_{is}(p,u_f), \quad s \in f\,, \end{aligned} \tag{2.3}$$

in an obvious notation.

First, assume that these cross-derivatives *exist and are continuous* —broadly that the corresponding short-run technologies are closed strictly convex bodies and that a small change in the endowment of fixed inputs does not lead to a large change in output[21]—and that the rank ρ of $G(p,u)$ is *constant* throughout an open product set

$$P \times U; \quad U = \bigtimes_f U_f\,, \tag{2.4}$$

in (p,u) space.

Second, I require that none of the controls be *specific to individual factories* f in the *firm* F. Even if its endowment u_f is held constant, that is, all ρ controls will be needed for the others. Hence the rank of the Jacobian matrix

$$G^{-f}(p,u_{-f}) = [g^{-f}_{is}(p,u_{-f})]_{s\notin f} \tag{2.5}$$

$$= [g_{is}(p,u)]_{s\notin f} \tag{2.6}$$

[20] I have not investigated the matter fully. It may be possible to get more general results.

[21] Because $g_{is} = \partial x_i/\partial u_s$ is assumed to exist. Continuity goes a little further.

is ρ throughout $P \times U$, where

$$g^{-f}(p, u_{-f}) = \sum_{g \neq f} g^g(p, u_g) = g(p, u) - g^f(p, u_f) \qquad (2.7)$$

is the gross profit function for the remaining factories.

Why is this important?

The basic argument in aggregation theory runs roughly as follows: Fix prices. Having done so, measure v_m, say, by the quasi rents it generates.[22] Increase these by a million dollars by increasing[23] the endowments u_f of a particular *factory* f. Observe the change δx in total output, noting that $p \cdot \delta x = 1$.[24] It has all occurred in f; hence $\delta x_f = \delta x$. Now revert to the previous position and change the endowment u_g of another firm $g \neq f$ instead, again by just enough to increase those quasi rents by a million dollars. The change δx will be the same as before, so that $\delta x_g = \delta x = \delta x_f$. Return to the original position. Increase these quasi rents by two million dollars by giving two successive gifts of new equipment to f, each worth a million dollars in these terms. In each case f will vary its production in the same way as g would have had it been given the million in question. But nothing has happened to g in the meanwhile: had it been given either gift, it would have raised its output by the same δx_g. Hence both δx_f's equal this δx_g and hence each other. The relevant Engle curve in f, and hence in all the firms, is therefore a straight line; these are parallel to each other at these prices; and that is true at any prices in P. These parallel lines correspond to the activities I have talked about so much.

There are difficulties in this argument. How do we arrange, for instance, that the other aggregates v_n, $n \neq m$, are held constant or, failing that, identify the changes due to the variation in v_m? In traditional theory, where there is a single aggregate for each class of fixed input or current good, this is easily done. Here we have no such simple structure to help us—hence my reliance on calculus methods and search for merely local results.[25] The problem nevertheless remains: each control must relate to at least two factories.

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[22] The quasi rents are the profits over and above those generated by a base equipment vector $\overline{u}$. Read on for some problems.

[23] The precise terms I have used are most appropriate for capital aggregation though not really misleading for other aggregates. If we are seeking to increase the amount of skilled labour used at a certain p, it may be appropriate either to increase or decrease the endowment of fixed inputs—'vary' might be a better word.

[24] The unit being a million dollars' worth at these prices.

[25] The localities—i.e. neighbourhoods—may, of course, be large: the results are exact, not approximations.

Let us summarize our assumptions by saying that the ***technologies are appropriately convex and smooth*** and that *just ρ general*[26] ***controls are needed*** throughout $P \times U$.

That rank condition can be weakened. If you look over the argument, you will see that I moved one firm at a time, holding the others at the baseline. I threatened to move them, too, but never actually did; if I had, it would have been by one unit only. In a calculus framework, that would be an infinitesimal jump. This suggests the following weaker condition:

> There is a $\overline{u} \in U$ such that $G(p, u_f, \overline{u}_{-f})$ and $G^{-f}(p, \overline{u}_{-f})$ have the same rank $\rho = \#M$, each $p \in P$, $u_f \in U_f$, $f \in F$.

In fact, this too can be considerably weakened, as will be seen in Corollary 3, but I will nevertheless assume it at the outset, together with the existence and continuity of each $g^f_{is}(\cdot)$ in the same region, also oversufficient. Summarize this by saying that the technology is sufficiently convex and smooth and requires just ρ general controls.

I am now in a position to state the main result.

Proposition. *If these conditions hold, there exist $\rho = \#M$ functions $a^m(\cdot)$, $2\#F$ functions $k^f(\cdot, u_f)$, $b^f(\cdot)$ about any point $\overline{p} \in P$, and for all $u \in U$, such that*

$$g^f(p, u_f) = k^f(a(p), u_f) - b^f(p), \quad \text{for each } f \in F, \qquad (2.8)$$

where

$$a(p) = (a^m(p))_{m \in M}. \qquad (2.9)$$

Proof: Choose a price vector $\overline{p} \in P$ and a particular firm $h \in F$. Then $G(p, \overline{u})$ has a column basis

$$(g'_m(p, \overline{u}))_{m \in M}, \text{ say}, \qquad (2.10)$$

chosen from $G^{-h}(p, \overline{u}_h)$ in a neighbourhood $N(\overline{p})$ of $\overline{p}$, where the prime still denotes the price gradient. Set

$$a^m(p) = g_m(p, \overline{u}), \text{ for each } m \in M; \quad a(p) = (a^m(p))_{m \in M}, \qquad (2.11)$$

to get

$$g'^f_s(p, u_f) = \sum_m \lambda^{sm} a'^m(p), \text{ say}, \ s \in f \in F, \ p \in N(\overline{p}). \qquad (2.12)$$

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[26] 'General' means not specific to any individual factory.

If the level sets

$$P(\alpha) = \{p \in P | a(p) = \alpha\} \tag{2.13}$$

were arc connected on $N(\overline{p})$, we could integrate (2.12) to get

$$g_s^f(p, u_f) = \gamma^s(a(p)), \text{ say, for each } s \in f \in F, \tag{2.14}$$

and would be well on the way to proving the theorem. Unfortunately they need not be on $N(\overline{p})$. However, I will construct a neighbourhood $N^*(\overline{p}) \subseteq N(\overline{p})$ of $\overline{p}$ on which they are arc connected, in the Appendix, so that (2.14) holds on it.

Now choose any factory $f \in F$. $G(p, u_f, \overline{u}_f)$ and $G^{-f}(p, \overline{u}_{-f})$ have the same rank ρ, each $p \in P$, $u_f \in U_f$. Hence[27]

$$\begin{aligned} g_s^{\prime f}(p, u_f) &= \sum_{t \in f} \mu^{st} g_t^{\prime -f}(p, \overline{u}_{-f}) \qquad (2.15) \\ &= \sum_m v^{sm} a^{\prime m}(p), \text{ for each } s \in f,\ u_f \in U_f,\ p \in N^*(\overline{p}), \end{aligned}$$

by (2.12). Integrating this in the same manner, we get

$$g_s^f(p, u_f) = k_s^f(a(p), u_f), \text{ say,}^{28} \tag{2.16}$$

and thus

$$g^f(p, u_f) = k^f(a(p), u_f) - b^f(p), \text{ for each } u_f \in U_f,\ p \in N^*(\overline{p}), \tag{2.17}$$

because we chose any $f \in F$, for each $f \in F$. That proves (2.8) and the Proposition.

Corollary 1. We can replace the rank condition by

$$R(G(p, u_f, \overline{u}_{-f})) = R(G^{-f}(p, \overline{u}_f)) =: \rho^f(p), \tag{2.18}$$

where R stands for 'rank of', for each $p \in P$, $u_f \in U_f$, $f \in F$, and get similar results.

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[27] The μ and v below are functions, of course: hence the superscripts. It does not matter what they are functions of.

[28] It is trivial that this is a derivative. Call it k^{fs}. Then $k_t^{fs} = g_{st}^f = k_s^{ft}$.

Hints: Set $u_f = \overline{u}_f$ to get $\rho^f(p) = R(G(p,\overline{u})) = \rho(p)$, say. Since the ρ's are finite integers, $\overline{\rho} = \max\{\rho(p)|p \in P\}$ is attained at $\overline{p}$, say, and, since $G(\cdot,\overline{u})$ is continuous, in a neighbourhood $N^{**}(\overline{p})$ of $\overline{p}$. Proceed as above until $P(\overline{\rho}) = \{p|\rho(p) = \overline{\rho}\}$ has been exhausted. We now run into difficulty. The rank of a matrix is the order of the largest minor whose determinant does not vanish. Determinants may vanish at isolated points, for instance, or along curves. We have therefore to confine our attention at the outset to *regular points* at and about which $\rho(p)$ is constant and then extend our results to the others by continuity.

Corollary 2. We can choose each $a^m(\cdot)$, $b^f(\cdot)$, $k^f(\cdot,u_f)$ to be conical.

The functions are clearly differentiable. We may take

$$b^f(p) = -g^f(p,\overline{u}_f) \tag{2.19}$$

if we wish; it is then a loss function and hence closed concave, whereas each $a^m(\cdot)$ is then a marginal profit function.

Corollary 3. $g(p,n) = k(a(p),u) - b(p)$, where

$$k(a,u) = \sum_f k^f(a,u_f), \qquad b(p) = \sum_f b^f(p). \tag{2.20}$$

Corollary 4. We can take

$$v = k'(a(p),u) = (\partial k/\partial a^m)_{m\in M}, \tag{2.21}$$

since then

$$\begin{aligned} x = f'(p,u) &= \sum_m k_m a'^m(p) - b'(p), \\ &= \sum_m v_m a'^m(p) - b'(p). \end{aligned} \tag{2.22}$$

Corollary 5. If so,

$$v = \sum v_f, \tag{2.23}$$

where

$$v_f = k'^f(a(p),u_f), \quad \text{for each } f \in F, \tag{2.24}$$

and

$$g^f(p,u_f) = p\cdot x_f = \sum_m a^m(p)v_{fm} - b^f(p), \tag{2.25}$$

$$g(p,u) = \sum_m a^m(p)v_m - b(p). \tag{2.26}$$

The interpretation of v_{fm} as the scale at which activity m is run in factory f is now clear, as is that of $a^m(p)$ as its gross profitability, which is the same in all the factories.

The other main claim in the Introduction was that, given appropriate convexity, these results could be interpreted in the primal in terms of profit centres. I will consider one example. Suppose $k^f(\cdot, u_f)$ is closed convex as well as conical, as in the paragraph following equation (1.14). Define

$$S^f(u_f) = \{v_f \mid a \cdot v_f \leq k^f(a, u_f)\}; \tag{2.27}$$

then

$$v_f = k'^f(a, u_f) \tag{2.28}$$

maximizes $a \cdot v_f$ on $S^f(u_f)$ by the basic theorem of duality. Of course, this is only a local result here, too, because $k^f(\cdot)$ is defined only in a neighbourhood $N^*(\bar{p})$ of $\bar{p}$.[29]

3. Some comments and disclaimers

Managers probably take over other firms to increase their sense of power or security and their socially acceptable pay—buccaneers in search of immediate cash to stay in the game. Taking over and selling off are often moves that come to mind to meet immediate contingencies; and, once consulted, the experts will naturally see reasons why they might work. That is not just a matter of self-interest: to a cobbler there is nothing like leather. Even under straightforward long-run profit maximization, ease of control is not the only parameter that is important. Nevertheless, ease of control is important; in that context, people often talk in terms of management by objectives and of profit centres. This essay suggests that quantitative objectives are misconceived, at least for the organization as a whole; appropriately normalized, they have shadow prices associated with them, which are better planning tools and, given appropriate convexity, they naturally lead to the location of appropriate profit centres and management structures.

Under imperfect competition, of course, one would have to deal with internal shadow prices instead of observed market prices and probably distinguish between physically identical products in different submarkets.[30] That does not seem to affect the argument, nor, I

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[29] I have not extended the technologies beyond the region in question.

[30] Physically identical activities in different locations might accordingly be economically different. Their physical identity might, nevertheless, make it easier to predict their distinct marginal profitabilities as well as, of course, simplifying the choice and training of factory and process managers.

think, do tax-distorted transfer prices, though I have not looked into that in detail.

That the central office's function in short-run planning should largely be confined to the prediction of market conditions as shown in appropriate broad price indices seems reasonable.[31] That these should relate to the short-run profitabilities of a few basic processes would fall in very well with its long-run role, as in planning investment, where it would concentrate on the investment costs of increasing capacity in different locations and in planning future takeovers and sales.

Firms made up in this way would also accumulate expertise in selecting, appraising, and promoting their managers.

Martin Weitzman has shown how important it may be in practical programming that the right quantities of the right goods should be available in the right place at the right time. This cannot be done by setting a few control variables or their shadow prices, and may require quite a different mode of operation.

Appendix

Lemma. *Using the assumptions of the Proposition in §2, there is a neighbourhood $N^*(\overline{p}) \subseteq N(\overline{p})$ of $\overline{p}$ within which the level sets*

$$P(\alpha) = \{p \in P \mid a(p) = \alpha\} \tag{A1}$$

are arc connected.

Take a row basis I for[32]

$$A(p) = [a_i^m(p)] = [g_{mi}(p, \overline{u})]_{m \in M}\,, \tag{A2}$$

and hence for $G(p, \overline{u})$ and each $G^{-f}(p, \overline{u}_{-f})$ in a neighbourhood $N^1(\overline{p}) \subseteq N(\overline{p})$, set

$$p_i = \begin{cases} q_i & \text{when } i \in I\,, \\ r_i & \text{when } i \notin I\,, \end{cases} \tag{A3}$$

and solve

$$a(q, r) = a(p) = \alpha \tag{A4}$$

in a neighbourhood $N^2(\overline{p}) \subseteq N^1(\overline{p})$ of $\overline{p}$ to get[33]

$$q = b(\alpha, r)\,. \tag{A5}$$

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[31] In the short run, most markets are pretty imperfect. I have not looked into this in any depth: doing so would turn on the extent to which a firm would wish to exploit such imperfections, given the effect on its long-run position.

[32] That corresponding to the original nonzero $\rho \times \rho$ determinant.

[33] By the implicit function theorem in each case.

Since $b(\cdot)$ is bicontinuous,[33] the set

$$C = \{(\alpha, r) \mid \alpha = a(q, r), \text{ for some } (q, r) \in N^2(\overline{p})\} \tag{A6}$$

is open. It contains $(\overline{\alpha}, \overline{r})$, where

$$\overline{\alpha} = a(\overline{q}, \overline{r}) = a(\overline{p}). \tag{A7}$$

Take a rectangular neighbourhood $D \subseteq C$ of $(\overline{\alpha}, \overline{r})$ and its image

$$\begin{aligned} N^*(\overline{p}) &= \{(q, r) \mid q = b(\alpha, r) \text{ for some } (\alpha, r) \in D\} \\ &\subseteq N^2(\overline{p}) \subseteq N^1(\overline{p}). \end{aligned} \tag{A8}$$

Since $b(\cdot)$ is bicontinuous, $N^*(\overline{p})$ is open. Since $N^*(\overline{p})$ contains $\overline{p}$, it is a neighbourhood of $\overline{p}$.

I am now ready to show that the level sets $P(\alpha)$ are indeed arc connected in $N^*(\overline{p})$.

To do so, take any $(q^*, r^*) \in N^*(\overline{p})$. Let

$$\alpha^* = a(q^*, r^*). \tag{A9}$$

Take any $(q^{**}, r^{**}) \in N^*(\overline{p})$ such that $a(q^{**}, r^{**}) = \alpha^*$, and construct a path

$$r(t) = (1-t)r^* + tr^{**}; \quad q(t) = b(\alpha^*, r(t)), \quad 0 \leq t \leq 1, \tag{A10}$$

connecting (q^*, r^*) and (q^{**}, r^{**}).

Since $(\alpha^*, r(t)) \in D$, its image $(q(t), r(t)) \in N^*(\overline{p})$. Since $b(\cdot)$ is differentiable, it traces a smooth arc. Since α^* was freely chosen,[34] it is indeed true that the $P(\alpha)$ are arc connected in $N^*(\overline{p})$.

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[34] Here $\alpha^* = a(q^*, r^*)$. (q^*, r^*) freely chosen from $N^*(\overline{p})$, implies (α^*, r^*) freely chosen from D.

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